

An Exponential $F(R)$ Dark Energy Model

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April 16, 2013

Abstract

We present an exponential $F(R)$ modified gravity model in the Jordan and the Einstein frame. We use a general approach in order to investigate and demonstrate the viability of the model. Apart from the general features that this models has, which actually render it viable at a first step, we address the issues of finite time singularities, Newton's law corrections and the scalaron mass. As we will evince, the model passes these latter two tests successfully and also has no finite time singularities, a feature inherent to other well studied exponential models.

Introduction

General Relativity is one of the most sound scientific descriptions of nature, regarding gravitational interactions in scales where gravity becomes important. Hence, it has become a powerful tool to describe local gravitational interactions and moreover to describe how the universe evolves as a whole. The astrophysical observations of the late 90's have put into a different perspective the way that we think the universe is evolving. According to these observations, the universe has undergone two accelerating phases. The first was the inflation period, while the second is the present epoch's acceleration. In reference to the latter, this cosmic acceleration is know as dark energy. The observational data for the present epoch (the new results of Planck telescope) suggest that the universe is described by the Λ CDM model. The main features of this model is that the universe is almost spatially flat, and consists of ordinary matter ($\sim 4.9\%$), cold dark matter ($\sim 26.8\%$) and dark energy ($\sim 68.3\%$), with the latter being the reason behind the present day acceleration.

One of the most recent attempts to successfully describe the dark energy is provided by the $F(R)$ modified theories of gravity (for informative reviews and very important papers on these theories see [1–4]), in the context of which, what changes drastically is not the left hand side of the Einstein equations, but the right hand side. In order the Friedmann-Robertson-Walker equations give an accelerating solution, the energy momentum tensor

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must contain a fluid with negative w , and this is achieved with these $F(R)$ theories. Particularly, early time inflation and the late time acceleration enjoy a unified description within the self-consistent theoretical framework some of these models provide [1–7]. In addition, some of these models are put into a string theory oriented context [8] and also quantum corrections have been calculated [9]. Connections to other gravitational extensions of general relativity, such as Gauss-Bonnet gravity, have also been pointed out in the literature [10] and moreover some exact solutions in strong gravitational background have been done in [11].

A very interesting and novel feature that most of the $F(R)$ theories of gravity have, is the appearance of finite time singularities in the physical parameters. There are several types of singularities depending on which physical parameters become singular. This finite type singularities can completely change our universe’s evolutionary history, leading to rather dramatic final states, known as Big Rip, or Little Rip etc. For an important stream of papers in regard to cosmological singularities and related issues, see [1, 6] and references therein.

With the $F(R)$ theories being a a generalization of general relativity, inevitably, these theories are confronted with the successes of general relativity. This entails a number of constraints that need to be satisfied, in order a promising modified gravity theory can be considered viable. These constraints are related to the local tests of general relativity and additionally to various cosmological bounds. The local tests are related to planetary and star formation tests. Moreover, each $F(R)$ theory formulated in the Jordan frame has a corresponding scalar-tensor gravitational theory in the Einstein frame, the scalarons of which have to be classical, so that the theory is quantum-mechanically stable.

In this present letter, we shall analyze in detail one exponential model which has some very appealing features in reference to the constraints we mentioned above. Exponential models have been thoroughly studied in references [3, 4, 7]. One of the most appealing features of the exponential models is that, in some of them, singularities are absent, while at the same time these models successfully pass all the tests these models are confronted with. The model we shall present has very interesting attributes, since it is free of singularities. Also we address the matter instability and Newton’s law corrections issues and as we will demonstrate, the model at hand passes these tests successfully.

This letter is organized as follows: In section 1 we briefly present the general features of $F(R)$ theories in the Jordan frame within the context of the metric formalism. In section 2 we study in detail the exponential model and also we investigate all the criteria which have to be satisfied in order a modified gravity model can be considered viable. In section 3 we present briefly an interesting functional resemblance of two $F(R)$ models (one of which is the one presented in this article), to the fermi distributions connected to Woods-Saxons potentials. The conclusions follow in the end of the article.

1 General Features of $F(R)$ Dark Energy Models in the Jordan Frame

In this section we shall give a brief description of the main features of $F(R)$ theories in the Jordan frame. For a much more detailed account consult references [1, 2]. The $F(R)$ modified gravity theories are described by the following four dimensional action:

$$\mathcal{S} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) + S_m(g_{\mu\nu}, \Psi_m), \quad (1)$$

with $\kappa^2 = 8\pi G$ and S_m the matter action of the matter fields Ψ_m . We shall focus on the metric formalism for our study and in addition we shall assume that the form of the $F(R)$ theory that we shall present is of the form $F(R) = R + f(R)$. Within the context of the metric formalism, by varying the action (1) with respect to $g_{\mu\nu}$, we obtain the following equations of motion:

$$F'(R)R_{\mu\nu}(g) - \frac{1}{2}F(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F'(R) + g_{\mu\nu} \square F'(R) = \kappa^2 T_{\mu\nu}. \quad (2)$$

In the above equation, $F'(R) = \partial F(R)/\partial R$ and $T_{\mu\nu}$ is the energy momentum tensor. The main idea behind the $F(R)$ modified gravity theories is that, what is actually modified is not the left hand side of the Einstein Equations, but the right. Indeed, the above equations of motion can be cast in the following form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{\kappa^2}{F'(R)} \left(T_{\mu\nu} + \frac{1}{\kappa} \left[\frac{F(R) - RF'(R)}{2} g_{\mu\nu} + \nabla_\mu \nabla_\nu F'(R) - g_{\mu\nu} \square F'(R) \right] \right). \quad (3)$$

Thus, the energy momentum tensor has another contribution coming from the term:

$$T_{\mu\nu}^{eff} = \frac{1}{\kappa} \left[\frac{F(R) - RF'(R)}{2} g_{\mu\nu} + \nabla_\mu \nabla_\nu F'(R) - g_{\mu\nu} \square F'(R) \right], \quad (4)$$

and this term is what actually models the dark energy. Taking the trace of equation (2) we obtain the following equation:

$$3\square F'(R) + RF'(R) - 2F(R) = \kappa^2 T, \quad (5)$$

with T being the trace of the energy momentum tensor $T = g^{\mu\nu} T_{\mu\nu} = -\rho + P$, and ρ, P are the energy density and pressure of the matter. This equation actually shows us that there is another degree of freedom underlying the $F(R)$ theories, materialized by the scalar field $F'(R)$. Consequently, equation (5) is actually the equation of motion of this scalar degree of freedom which is called the “scalaron”. In a flat Friedmann-Lemaitre-Robertson-Walker spacetime, the Ricci scalar is given by:

$$R = 6(2H^2 + \dot{H}), \quad (6)$$

with H the Hubble parameter, and the “dot” represents differentiation with respect to time. Accordingly, the cosmological dynamics are governed by the following equations:

$$\begin{aligned} 3F'(R)H^2 &= \kappa^2(\rho_m + \rho_r) + \frac{(F'(R)R - F(R))}{2} - 3H\dot{F}'(R), \\ -2F'(R)\dot{H} &= \kappa^2(p_m + 4/3\rho_r) + F\ddot{F}'(R) - H\dot{F}'(R), \end{aligned} \quad (7)$$

where ρ_r and ρ_m stand for the radiation and matter energy density respectively. Hence, the total effective energy density and pressure of matter and geometry are [1, 4]:

$$\begin{aligned} \rho_{eff} &= \frac{1}{F'(R)} \left[\rho + \frac{1}{\kappa^2} \left(F'(R)R - F(R) - 6H\dot{F}'(R) \right) \right] \\ p_{eff} &= \frac{1}{F'(R)} \left[p + \frac{1}{\kappa^2} \left(-F'(R)R + F(R) + 4H\dot{F}'(R) + 2\ddot{F}'(R) \right) \right], \end{aligned} \quad (8)$$

with ρ, P the total matter energy density and pressure respectively.

2 General Study and Detailed Study of the Exponential Model

In principle, every viable $F(R)$ model in the metric formalism has to satisfy certain conditions, in order to be compatible with observations and also consistent with theoretical predictions. In this section we shall present a viable exponential model, which has as limiting cases other exponential models that exist in the literature, and we shall thoroughly study the quantitative features of the $F(R)$ theory it describes. The exponential model is of the following form,

$$F(R) = R - \frac{C}{A + Be^{-R/D}} + \frac{C}{A + B}, \quad (9)$$

with A, B, C, D constants. In the rest of this paper, we shall denote $f(R)$, the second term on the right hand side of Eq. (9), that is:

$$f(R) = -\frac{C}{A + Be^{-R/D}} + \frac{C}{A + B}. \quad (10)$$

2.1 General Features of the Model

Let the Ricci scalar of the universe at the present epoch be denoted as $R_0 \simeq 10^{-66} \text{eV}^2$ and additionally denote as Λ, Λ_I the cosmological constant of the universe today and during the inflation period of the universe, respectively. The general features of a viable $F(R)$ model are the following [1]:

- (i) $F'(R) > 0$ for $R \geq R_0$. Moreover if R_1 is a de Sitter final attractor of the system, then this inequality has to hold true for $R \geq R_1$.
- (ii) $F''(R) > 0$ for $R \geq R_0$. This restriction is required for consistency with local gravity tests, for the presence of a matter dominated epoch and for the stability of cosmological perturbations.

- (iii) $F(R) = R - \Lambda_I$ for $R \rightarrow \infty$. This is required in order inflation occurs in the universe.
- (iv) $F(R) = R - \Lambda$ for $R \rightarrow R_0$ and $f(R_0) = -\Lambda$. This is required in order the late time acceleration occurs.
- (v) $F(0) = 0$, in order a flat spacetime solution exists.

In order model (9) satisfies the above constraints, the constants have to satisfy some conditions. In order to have a clear picture of how the derivatives behave as a function of R , we quote them below:

$$F'(R) = 1 - \frac{BCe^{\frac{R}{D}}}{D\left(B + Ae^{\frac{R}{D}}\right)^2}, \quad (11)$$

$$F''(R) = \frac{BCe^{\frac{R}{D}}\left(-B + Ae^{\frac{R}{D}}\right)}{D^2\left(B + Ae^{\frac{R}{D}}\right)^3}.$$

Due to the square in the denominator of both the derivatives $F'(R)$ and $F''(R)$, it is not very difficult to choose the constants A, B, C, D . Indeed, by looking both expressions, the first and second derivative are always positive when $A > B$ and $D > C$. In order not to fall into inconsistencies, we also require that the inequality means that the related constants have at least $\sim 10\%$ difference in their scale, which for example means that A is ten times larger than B . Further restrictions on the constants shall be imposed later on in this section. Hence, the conditions (i) and (ii) in the list above are satisfied for all the values of the Ricci scalar R , an argument that is further supported from the fact that the denominator behaves as $\sim e^{2R/D}$, rendering the fraction smaller than one for all R , in reference to the first derivative. The second derivative is always positive due to the fact that the term $-B + Ae^{\frac{R}{D}}$ is always positive for $A > B$ due to the exponential term. Condition (v) is automatically satisfied for the model (9).

When $R \rightarrow \infty$, the function $F(R)$ behaves as:

$$F(R) \sim R - \frac{CB}{A(A+B)}. \quad (12)$$

It is obvious that Λ_I , the early time cosmological constant that governs the inflationary expansion of the universe, must be $\Lambda_I \sim \frac{CB}{A(A+B)}$. Consequently, since $\Lambda_I \sim 10^{20-38} \text{eV}^2$, the values of A, B, C have to be chosen appropriately in order this requirement is satisfied. A set of values that will prove to be appropriate, in order the present and other constraints that will be imposed later on in this section are satisfied, are the following:

$$A = 0.1, B = 0.01, D = 10^{22}, C = 10^{21}. \quad (13)$$

Using the values (13), the inflation period cosmological constant is approximately $\Lambda_I \sim 9.09 \times 10^{20}$. After checking these very general features, we proceed to more elaborate criteria for the viability of the present exponential model.

As a final comment before we proceed, notice that when $R \gg R_0$, the model (9), is approximately equal to:

$$F(R) \sim R - \frac{C}{A} + \frac{BC}{A^2} e^{-\frac{R}{B}} + \frac{C}{A+B} \quad (14)$$

Therefore we can see that the model of [3, 4] and the model (9) coincide in the large curvature limit. Therefore we expect that these models may give rise to similar cosmological dynamics. We shall elaborate on this issue later on in this section.

2.2 Scalon Mass and Effective Potential

The approach we adopt in the present section is based on references [3, 4]. Following [3, 4], the scalaron dynamical degree of freedom is governed by the trace equation of motion (5). By making the substitution $F'(R) = 1 + f'(R) = e^{-\chi}$, and performing a perturbation around a constant scalar curvature solution R_* , so that $R = R_* + \delta R$, the equation of motion of the scalaron field read:

$$\square \delta \chi - \frac{1}{3} \left(\frac{1 + f'(R_*)}{f''(R_*)} - R_* \right) \delta \chi = - \frac{\kappa^2}{6(1 + f'(R_*))} T, \quad (15)$$

with $\delta \chi$ being related to δR as follows:

$$\delta R = - \frac{1 + f'(R_*)}{f''(R_*)} \delta \chi. \quad (16)$$

The constant scalar curvature solution R_* , is a solution to the equation:

$$R_* + 2f(R_*) - R_* f'(R_*) = 0. \quad (17)$$

The mass square of the scalaron plays an important role in reference to local and planetary test of the modified gravity theory. This is equal to:

$$M^2 = \frac{1}{3} \left(\frac{1 + f'(R_*)}{f''(R_*)} - R_* \right). \quad (18)$$

Only a positive and large value of the mass square is required, in order no tachyonic instability occurs and the corrections to the Newton's law are small. Substituting the model (10), in Eq. (17), we have that R_* is a de Sitter solution of the system. The value $R_* = 0$ corresponds to the Minkowski spacetime, so we would like to see if Minkowski spacetime is stable. Particularly, we shall examine if the scalaron theory perturbed around the Minkowski solution $R_* = 0$ is stable, or if it has tachyonic or any other sort of instabilities. This test is critical for flat spacetime local tests. For $R_* = 0$ and using the values (13), the mass of the scalaron field for the model (10), is equal to:

$$M^2(0) \sim 4.552 \times 10^{22}. \quad (19)$$

The above value is positive and consequently there is no tachyonic instability. In addition, since it is very large, the δR perturbations at long ranges tend to zero and therefore,

Newton's law has no considerable corrections. Actually, the mass square is positive for a large range of values of the parameters, provided that $A > B$ and $D > C$. This can be easily seen by looking the analytic expression of the mass, for the model at hand:

$$M^2(0) = \frac{(A+B)D(-BC + (A+B)^2D)}{3(A-B)BC}. \quad (20)$$

Before closing this issue, we shall search for other de Sitter solutions of Eq. (17). We are especially interested in a late time de Sitter point. Following the notation of [4], let $G(R) = F(R) - RF'(R)$. Since $G'(0) < 0$ (see Eq. (11)), the function becomes negative and increases after $R = R_0$. For $R = \mathcal{O}(\Lambda)$, the derivatives of the $F(R)$ function behave as:

$$F'(R) \simeq 1, \quad F''(R) \simeq 0. \quad (21)$$

The computation of the late time de Sitter point is straightforward, since Eq. (17), simplifies to the following:

$$2F(R) = R. \quad (22)$$

Searching a solution around $R = \Lambda$ and using the values (13) for the variables A, B, C, D , we easily get numerically, that $R = \Lambda$ is actually a late time de Sitter solution.

2.3 Matter Instability Analysis of the Exponential Model

We now turn our focus on the matter instability issue, which might occur when the scalar curvature is large, in reference to the one corresponding to the present epoch. The scalaron equation can be written in the form [3, 4]:

$$\square R + \frac{f'''(R)}{f''(R)} \nabla_\rho R \nabla^\rho R + \frac{(1 + f'(R))R}{3f''(R)} - \frac{2(R + f(R))}{3f''(R)} = \frac{\kappa^2}{6f''(R)} T. \quad (23)$$

We perturb the scalar curvature around an Einstein gravity solution $R_e = -\frac{\kappa^2 T}{2} > 0$, so that $R = R_e + \delta R$, and we obtain the equation:

$$(-\partial_t^2 + U(R_e))\delta R + \text{const} \simeq 0, \quad (24)$$

with $U(R_e)$ the effective potential, which is:

$$U(R_e) = \left(\frac{F''''(R_e)}{F''(R_e)} - \frac{F''''(R_e)^2}{F''(R_e)^2} \right) \nabla_\rho R_e \nabla^\rho R_e + \frac{R_e}{3} - \frac{F'(R_e)F''''(R_e)R_e}{3F''(R_e)^2} - \frac{F'(R_e)}{3F''(R_e)^2} + \frac{2F(R_e)F''''(R_e)}{3F''(R_e)^2} - \frac{F''''(R_e)R_e}{3F''(R_e)^2}. \quad (25)$$

In the case the effective potential is positive, the perturbation δR becomes exponentially large and the whole system is rendered unstable [3, 4]. Therefore, the matter stability condition is $U(R_e) < 0$. Let us see what happens in the case of the exponential model

at hand (9). If we substitute (9) to Eq. (25), and keep only the leading order terms, the potential $U(R)$ can be written as follows:

$$U(R_e) \sim \frac{D^2 e^{-\frac{R_e}{D}} \left(B + A e^{\frac{R_e}{D}} \right)^3}{3BC \left(B - A e^{\frac{R_e}{D}} \right)}. \quad (26)$$

Due to the term in the denominator, namely $B - A e^{\frac{R}{D}}$, and since we imposed the condition $A > B$ for the viability of the model, the potential given by relation (26) is always negative, for all curvature values and for all the values of the parameters so that, $A > B$ and $D > C$. Thereby, the matter stability condition is fulfilled for the model (9).

Before finishing this section we would to address an issue related to the matter era period of the cosmological model. Following [4], during the matter era period, and neglecting the contribution of radiation, one has:

$$\rho_{eff} = \frac{\rho}{F'(R)}, \quad \frac{RF'(R)}{F(R)} = 1, \quad (27)$$

with ρ_{eff} and ρ the total energy density (matter and dark energy) and ρ the matter energy density. From the second relation of Eq. (27) we get [4]:

$$\frac{F''(R)}{F'(R)} = 0. \quad (28)$$

So at the matter era we must have that $F''(R) \simeq 0$. As pointed in [4], an $F(R)$ -theory is acceptable if the modified gravity contribution vanishes during this era and also $F'(R) \sim 1$. In addition, the second derivative must be very small and positive [4]. Let us examine what happens for the model (9). As can be seen, the second derivative of the $F(R)$ exponential is governed by the exponential term in the denominator, that is:

$$F''(R) \simeq \frac{BC}{D^2 A^2 e^{\frac{R}{D}}}. \quad (29)$$

Using the values (13) for the parameters A, B, C, D and due to the exponential dependence, it can be easily checked that the second derivative during the matter era is very close to zero and positive.

2.4 Einstein Frame Analysis of the Model

Apart from the latter approach we adopted in order to address the problem of matter instability, there is another approach for this problem which is related to the Einstein frame of the $F(R)$ theory. Hence, we shall investigate the problem of matter instability and Newton's law corrections, in the Einstein frame. The Jordan frame and the Einstein frame description of $F(R)$ modified theories of gravity are mathematically equivalent. Actually, the $F(R)$ theories in the Jordan frame become scalar tensor theories with a

potential in the Einstein frame [1, 3, 4, 6]. Following reference [3], an auxiliary field \mathcal{A} is introduced in the action (1)

$$\mathcal{S} = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left((R - \mathcal{A})(1 + f'(\mathcal{A})) + \mathcal{A} + f(\mathcal{A}) \right). \quad (30)$$

Using the conformal transformation $g_{\mu\nu} \rightarrow e^\sigma g_{\mu\nu}$, with $\sigma = -\ln(1 + f'(\mathcal{A}))$, and bearing in mind that the equations of motion with respect to \mathcal{A} , yield the result that $\mathcal{A} = R$, we obtain the Einstein frame action:

$$\mathcal{S}_E = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{3}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right), \quad (31)$$

with the effective potential $V(\sigma)$, being equal to:

$$V(\sigma) = e^\sigma g(e^{-\sigma}) - e^{2\sigma} F(g(e^{-\sigma})) = \frac{\mathcal{A}}{F'(\mathcal{A})} - \frac{F(\mathcal{A})}{F'(\mathcal{A})^2} \quad (32)$$

The mass corresponding to the field σ is equal to:

$$m_\sigma^2 = \frac{1}{2} \frac{d^2 V(\sigma)}{d\sigma^2} = \frac{1}{2} \left(\frac{\mathcal{A}}{F'(\mathcal{A})} - \frac{4F(\mathcal{A})}{F'(\mathcal{A})^2} + \frac{1}{F''(\mathcal{A})} \right). \quad (33)$$

In order the Newton's law corrections are small, this mass term has to be a large number and of course positive in order tachyonic instabilities are avoided. Let us see how this term behaves. Substituting Eq. (9) into (33), the mass of the σ field is equal to (we keep only dominating terms):

$$m_\sigma^2 \simeq \frac{D^2 A^2 e^{\frac{2R}{D}}}{2BC} \quad (34)$$

Let us find the value of the above mass, by using the values (13) for the parameters. For a wide range of values of the scalar curvature, the mass (34) has the value $m_\sigma^2 \sim 7.2 \times 10^{22}$, which is large, and thus we conclude that the Newton's law corrections are negligible. In addition, since it is positive, the theory is free from tachyonic instabilities.

2.5 Analysis of Finite Time Singularities of the Exponential Model

In this section we shall analyze in detail the singularity structure of the model at hand. The singularities that quite frequently occur in $F(R)$ theories of gravity have the form $H(t) = h/(t_0 - t)^\beta$, with h and t_0 positive constants [4]. There are four types of finite time future singularities [1, 4, 6] which are listed below:

- Type I (Big Rip): For $t \rightarrow t_0$, $a(t) \rightarrow \infty$, $\rho_{eff} \rightarrow \infty$ and $|p_{eff}| \rightarrow \infty$ ($\beta \geq 1$)
- Type II (sudden): For $t \rightarrow t_0$, $a(t) \rightarrow a_0$, $\rho_{eff} \rightarrow \rho_0$ and $|p_{eff}| \rightarrow \infty$ ($-1 < \beta < 0$)
- Type III: For $t \rightarrow t_0$, $a(t) \rightarrow a_0$, $\rho_{eff} \rightarrow \infty$ and $|p_{eff}| \rightarrow \infty$ ($0 < \beta < 1$)
- Type IV: For $t \rightarrow t_0$, $a(t) \rightarrow a_0$, $\rho_{eff} \rightarrow 0$ and $|p_{eff}| \rightarrow 0$ and higher derivatives of the Hubble parameter H diverge.

The singularity structure of the $F(R)$ model (9) is quite similar to the one presented in reference [4]. Indeed, at the limit $R \rightarrow \infty$, we have:

$$\lim_{R \rightarrow \infty} F(R) \simeq R + \text{const}, \quad \lim_{R \rightarrow \infty} F'(R) \simeq 1. \quad (35)$$

In addition, the higher order derivatives of the $F(R)$ function (9), tend to zero exponentially. Therefore, neither Type I nor Type III singularities can appear in the present model, which is exactly what happens in reference [4]. Now let us consider the other two possible singularities, namely Type II and Type IV. In reference to the first, when $R \rightarrow -\infty$, the $F(R)$ function can be written as:

$$F(R) \simeq R - \frac{C}{B} e^{-R/D} + \frac{C}{A+B}. \quad (36)$$

Consequently, the total effective energy density ρ_{eff} of Eq. (8), exponentially decays for the model (9) and also the total effective pressure behaves analogously. Thereby, the Type II singularity cannot occur, since the pressure does not diverge. In addition, Type IV singularities cannot be realized. The reason is in absolute concordance with the arguments of [4]. Indeed, when $R \rightarrow 0$, the $F(R)$ function behaves as

$$F(R) \simeq R - \frac{BC}{(A+B)^2 D} R, \quad (37)$$

and therefore, the effective energy density behaves as $\sim 1/(t_0 - t)^{\beta+1}$ and is larger than $1/(t_0 - t)^{2\beta}$, for $\beta < -1$. Hence the Type IV singularity cannot occur. As in the [4] case, the model we presented is free of singularities. The only difference between the two models is the different reasoning for the Type II singularity, since in the present model it is the finite pressure that renders the model free of these singularities, in contrast to the model of [4], where the effective energy density diverges. In addition, since according to Eq. (14), the two models overlap when $R \rightarrow \infty$, the reasoning on why the Type I and Type II singularities do not occur in both models is identical in both cases, as it was expected.

3 Functional Resemblance of two $F(R)$ Models to Woods-Saxons Nuclear Potentials

It worths to note that the model we used in this letter described by the $F(R)$ function (9), has many similarities to some Woods-Saxons potentials that are used to describe the Nucleon interactions. The Woods-Saxon potentials are phenomenological potential for the nucleons in the atomic nucleus. It is used in the shell model and describes the forces applied to each nucleon. The general form of the potential is,

$$V(r) = -\frac{V_0}{1 + e^{\frac{r-b}{\alpha}}}, \quad (38)$$

where r the distance from the center of the nucleus. Note one particular fermi function distribution, related to (38)

$$u_f(r) \simeq B_c \tanh\left(\frac{-r}{2\alpha}\right). \quad (39)$$

Note that this function (39) has great similarities to the Tsujikawa model [1, 2], for $F(R)$ modified gravitational theories, namely:

$$F(R) = R - R_c \tanh(R/R_c). \quad (40)$$

Indeed, if we change $r \rightarrow -R$, and add R in expression (39) we get the Tsujikawa model. Using the same line of reasoning, the following Fermi shape distribution,

$$u_f(r) = \frac{1}{A + B e^{\frac{r}{\alpha}}}, \quad (41)$$

leads to the model we used in this article. Of course there is no physical connection between the two systems, the nucleus potential and the gravitational systems. We wanted to stress the fact that by performing the same transformations to two different nuclear fermi distributions, we get two viable $F(R)$ dark energy models.

Concluding Remarks

In this article, we studied in detail an exponential model of $F(R)$ gravity in the metric formalism. Apart from the very general features that make the model viable at a first step, the model has also other interesting features that render it promising. Particularly, the scalaron mass is positive and large, which means that the theory is free of tachyonic instabilities and free of matter instabilities. The matter instability and Newton's law corrections problem has been addressed in both the Jordan and Einstein frames, and the result verified once more that the model does not lead to problems or inconsistencies. Moreover, we studied the finite time singularities issue and as we demonstrated, the model is free of the four types of finite time singularities. In addition, we also presented what motivated us to use such an exponential model. As we showed, this model and another $F(R)$ model have functional resemblance to two Fermi distributions corresponding to Woods-Saxons potentials.

We intended to show the basic features of this $F(R)$ model, using a very general approach, similar to the one used in the articles [3, 4]. It is interesting to address other interesting issues, such as oscillations of the dark energy, or curvature singularities in dense gravitational backgrounds [12], which however are beyond the scope of this letter. We hope to address such issues in a future work.

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